Design of Fibre-Reinforced Concrete Structures

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Abstract: Different types of concrete are invented in the past years, in order to improve it`s behaviour for different loading and environmental situations. Among them, fibre reinforced concrete (FRC) is one of the most recently developed concrete types. Design procedures for the FRC elements are included in only a few existing code provisions. Among them, fib Model Code 2010 (MC 2010) provides the biggest amount of necessary information and recommendations in order to design FRC elements. Yet, a lack of the guidelines for the design of elements loaded with different combinations of bending moment and axial force is noticed in the existing code provisions. Therefore, in the scope of this paper, interaction curves for FRC are developed. The interaction curves are developed for both, ultimate limit state (ULS) and serviceability limit state (SLS). Furthermore, example of the use of such interaction curves is given. Comparison in the design of reservoir made of reinforced concrete (RC) and FRC is performed. It is shown that the use of FRC especial benefits design according to SLS, as FRC provides better behaviour regarding crack spacing and crack width of element, while developed interaction curves for SLS significantly decrease the time necessary for such a design.

Index Terms: ***FRC, fibre, interaction curve, fib Model Code 2010, reservoir design***

1. *INTRODUCTION*

CONCRETE is the second most consumed material in the world, after water [1]. Hence, necessity for improving the different properties of concrete led to the development of various concrete types, such as Recycled Aggregate Concrete (RAC), Self Compacting Concrete (SCC), Fibre Reinforced Concrete (FRC), etc. Use of the fibres in order to improve the concrete properties dates back to the ancient times [2]. In the modern times, the first use of FRC is related to the second half of the 19th century. After the series of the conducted tests in the 20th century [3,4], FRC is patented as a construction material. FRC is defined as a mixture of concrete matrix and fibre. Fibre can be made of different materials, such as steel, glass, carbon, polypropylene etc, and can significantly vary in the length and diameter. Development of plastificators enhanced the use of FRC, by facilitating workability of such a mixture. The main advantage of FRC in comparison to plain concrete, represent its residual tensile strength, as shown in Figure 1 [5]. Therefore, FRC is often used for the construction of the prefabricated tunnel segments, as these elements are under pure compression during the exploitation, but can exhibit flexural tensile stresses during the storage and transport. Moreover, FRC is employed for the construction of pavements, reservoirs, roofs, facades, etc.

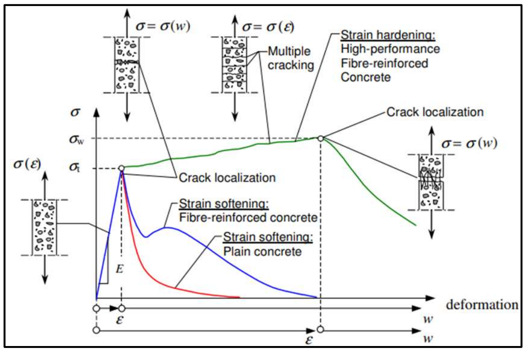


Figure 1. Comparison of RC and FRC in tension

2. *PROBLEM STATEMENT*

The biggest progress in the understanding of the mechanical and post-peak behaviour of FRC is made in the last 20 years. This is followed by the expansion in the use of FRC in the construction practice. Yet, design procedure of FRC elements is implemented in only few of the existing code provisions. In such a provisions, description of FRC under flexural bending, compression, tension, shear and different loading combinations is given. Nevertheless, significant lack is noticed in the design of the elements under combined axial force and bending moment (alternative direction), as no interaction curves are provided. Therefore, the investigation of this article is headed in the direction of the development of the interaction curves of FRC, according to the constitutive relationships from fib

Model Code 2010 [6]. Derived interaction curves are used in order to design reservoir and to compare such a design with the usual reinforced concrete. Structural analysis of reservoir is performed in Autodesk Robot software [7], and the data are then used as an input for the developed interaction curves for FRC.

3. *THE BEST EXISTING SOLUTIONS*

Mechanical behaviour and cross-section design of FRC elements is given in a few existing code provisions, such as RILEM TC 162-TDF [8]; EHE – 08 [9]; CNR-DT [10]. The latest regulation that includes the analysis of FRC is fib Model Code (MC 2010) [6]. In the MC 2010, detailed cross-section analysis of FRC elements is given. Analysis of the cross-section according to the MC 2010 of FRC is given in Figure 2.

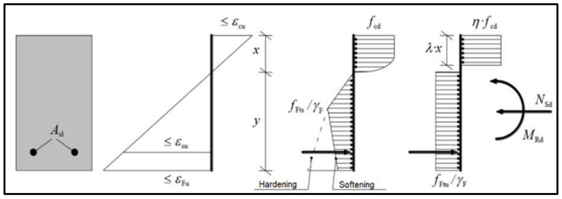


Figure 2. Cross-section analysis of FRC

The main difference in comparison to reinforced concrete is the residual tensile strength for serviceability limit state f_{fts} and ultimate limit state f_{ftu}. These parameters represent the contribution of the fibre on the behaviour of FRC under tensile regime. Values of these parameters can be calculated using Equations:

$$f_{Fts} = 0.45 * f_{R1} \tag{1}$$

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3} * (f_{Fts} - 0.5 * f_{R3} + 0.2 * f_{R1}) \tag{2}$$

Where f_{R1} and f_{R3} represent residual tensile strength at 0.5 and 2.5 mm of crack mouth opening displacement (CMOD) according to the test procedure and w_u represent the value of CMOD (Figure 3).

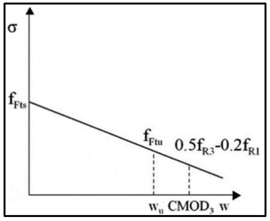


Figure 3. Tensile stresses in FRC fibre as a function of the crack width

Under combined bending moment and axial force, FRC cross-section can be in the range of small or large eccentricity. In the case of large eccentricity, neutral axis is in the cross-section, hence part of the cross-section is under compressive, while other part under tensile stresses. Cross-section analysis of FRC in the case of large eccentricity is shown in Figure 4. It is assumed that compressive strain in the most compressed fibre reached the limit of 3.5 ‰ [6]. As the limiting value of the crack width is 2.5 mm, maximal tensile strain is equal to the minimum of two cases: ε_{c1} = min {20 ‰ ; w_u/l_{cs}), where l_{cs} is the characteristic length and it is equal to the depth of the tensile part of the cross-section [6].

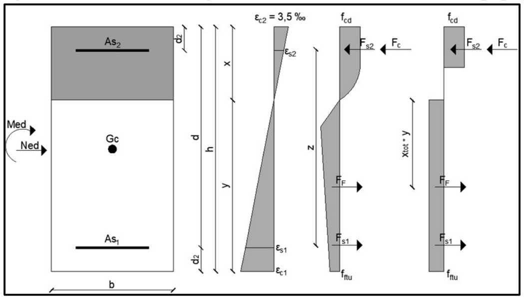


Figure 4. FRC cross-section in the area of large eccentricity

Once the cross-section is in the area of small eccentricity and it is completely compressed due to the combination of the bending moment and axial force, strains in the concrete are in the range from 2 do 3.5 ‰ (Figure 5). Usually, in this situation cross-section is simmetrically reinforced.

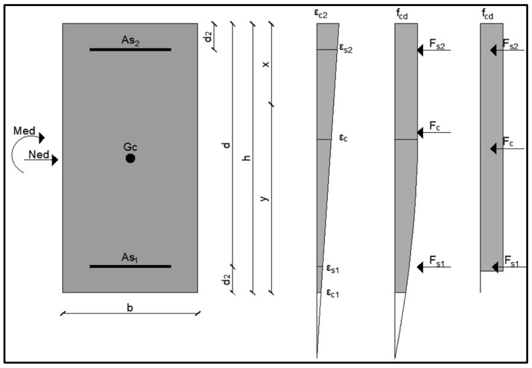


Figure 5. FRC cross-section loaded with bending moment and compressive force (fully compressed section)

If the cross-section is in the area of small eccentricity and it is loaded by bending moment and axial tensile force, three different cases for the ULS analysis can be distinguished : i) Tensile flexural strain is equal to 20 ‰, while axial tensile strain is equal to 10 ‰ (Figure 6); ii) Tensile flexural strain is between 10 and 20 ‰, while axial tensile strain is equal to 10 ‰ (Figure 7); iii) Both tensile flexural strain and axial tensile strain are lower than 10 ‰, due to the limitation of the crack width in FRC (Figure 8) [6].

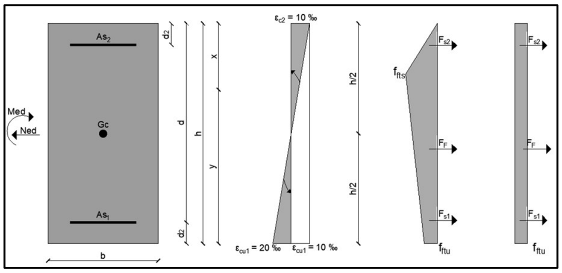


Figure 6. FRC cross-section loaded with bending moment and axial force (flexural strain is 20 ‰, while axial tensile strain is 10 ‰)

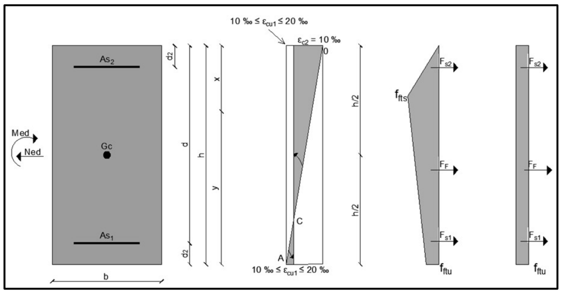


Figure 7. FRC cross-section loaded with bending moment and axial force (flexural strain is between 10 and 20 ‰, while axial tensile strain is 10 ‰)

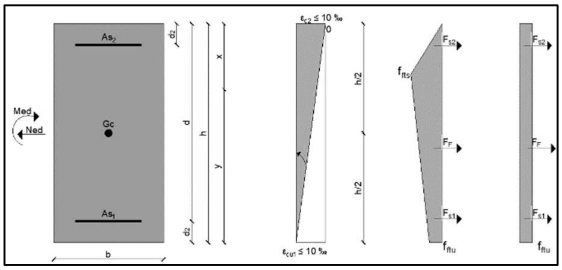


Figure 8. FRC cross-section loaded with bending moment and axial force (flexural and tensile strain are lower than 10 ‰)

For all mentioned situations, unknown values of some combination of axial force and bending moment can be calculated by writing the equilibrum equation of the axial forces and bending moment around the tensile reinforcement.

4. *THE PROPOSED SOLUTION*

In this paper, the existing constitutive laws for FRC reported in fib MC 2010 [6] and described in Chapter 3 are used in order to analyse the behaviour of FRC. Furthermore, cross-section analysis and stress-strain relationships from MC 2010 are used in order to derive the interaction curves. In order to develop the interaction curves, situations in which neutral axis is in the cross-section or outside of the cross-section, are analyzed (small and large eccentricity of the axial force). The interaction curves are developed for both, serviceability and ultimate limit states (SLS and ULS), hence the complete design of FRC cross-section can be performed using them. ULS interaction curves are derived by assuming either the compressive failure of FRC, or tensile failure of reinforcement or FRC. SLS interaction curves are derived in the similar manner, but limiting the stress in the concrete to 60 percent of the characteristic value (f_{ck}) and assuming linear analysis[6]. Therefore, maximal compressive strain of FRC in SLS analysis is:

$$\varepsilon_{cmax} = 0.6 * \frac{f_{ck}}{E_{cm}} \tag{3}$$

5. *CONDITIONS OF THE ANALYSIS TO FOLLOW*

Once the cross-section is loaded by the combination of bending moment and axial force, it is necessary to calculate the equilibrium equations for every possible state of strains. In order to optimize the calculations, interaction curves are made. Interaction curves are used once the dimension of the cross-section are known, or once the ratio of height / width of the cross-section is known. Furthermore, for RC, interaction curves are used in order to adopt the necessary amount of the reinforcement, in order that ULS analysis is satisfied. For FRC, beside above mentioned parameters of the cross-section, residual tensile strength f_{R1} and f_{R3} of FRC are an input parameter. The other option is that for the adopted characteristics of the cross-section and adopted reinforcement, values of the residual tensile strength of FRC are varied in order to satisfy the limit states. Hence, by using the interaction curves, it is possible to design the cross-section for all the possible combinations of bending moment and axial force. In such a manner, calculation time for the design of FRC cross-section is highly reduced. Use of the proposed stress–strain relationships and cross-section analysis from MC 2010 allows to design the cross section for both, ULS and SLS. Once the cross-section parameters, amount of the reinforcement, concrete grade and fibre parameters are known, it is possible to calculate axial capacity (N_{ed}) and bending moment capacity (M_{ed}) of the cross-section. As mentioned, this is done by writing the equilibrium equations of normal force and bending moment. This procedure is necessary to conduct for the sufficient number of strain pairs at the top and bottom edge of the cross-section, in order to construct the interaction curves with satisfying accuracy.

6. *DETAILS OF THE PROPOSED SOLUTION*

6.1 Interaction curves for ULS

In order to construct the interaction curves with satisfying accuracy, the minimum is to include all the strain pairs, which represent the limiting values between the different states of the cross-section, described in Chapter 3. The limiting strain pairs for the different state of the cross-section are shown in Table 1.

Cross-section state	Lower strain pair limit	Upper strain pair limit
1.) Small eccentricity and tensile force	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{ud} / \varepsilon_{ud}$	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{ud} / 0$
2.) Large eccentricity and pure bending	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{ud} / 0$	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{ud} / \varepsilon_{cu2}$
3.) Large eccentricity and pure bending	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{ud} / \varepsilon_{cu2}$	$(\varepsilon_{s1} / \varepsilon_{c,2})$ = $\varepsilon_{yd} / \varepsilon_{cu2}$
4.) Large eccentricity and pure bending	$(\varepsilon_{s1} / \varepsilon_{c,2})$ = $\varepsilon_{yd} / \varepsilon_{cu2}$	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $0 / \varepsilon_{cu2}$
5.) Small eccentricity and compressive force	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $0 / \varepsilon_{cu2}$	$(\varepsilon_{c,1} / \varepsilon_{c,2})$ = $\varepsilon_{c2} / \varepsilon_{c2}$

Table 1. Limiting values of strains in FRC cross-section for different section states

Where, ε_{ud} is the maximal strain in the reinforcement, ε_{yd} is the yielding strain of the reinforcement, ε_{c2} is the maximal compressive strain of FRC of 3.5 ‰, ε_{cu2} is the compressive strain for the cross-section in pure compression and is equal to 2 ‰. Furthermore, ε_{c1}, ε_{c2} and ε_{s1} are strains at the bottom and top edge of the cross-section and in the reinforcement, respectively. For each of the possible strain pairs, one point in the Cartesian coordinate system is created, with the pair values of N_{ed} and M_{ed}. Drawing the lines that connect all the calculated points on the graph, interaction curve is created. Such a created interaction curve represents a combined capacity of the cross-section. Hence, the area limited by the curve and axis, represents the area of all the possible capacity states. If the point which is defined by pair of N_{ed} and M_{ed} is outside of such a defined area, failure of the cross-section will occur. In such a situation, it is necessary to adopt either new dimensions of the cross-section, amount of the reinforcement or FRC parameters. Described procedure can be repeated, but with different amount of the reinforcement. Hence, a new curve is defined on a graph. The same process can be repeated as many times as necessary, in order to derive group of the curves for different ratios of the reinforcement / cross-section area. It is important to mention that such a derived group of interaction curves is valid only for the adopted dimensions of the cross-section, adopted parameters of FRC and reinforcement and fibres. In order to create interaction curves independent from the cross-section dimensions and material parameters, dimensionless coefficients are introduced:

$$\mu_{Rd} = \frac{M_{rd}}{b * h^2 * f_{cd}} \tag{4}$$

$$\nu_{Rd} = \frac{N_{rd}}{b * h * f_{cd}} \tag{5}$$

N_{rd} and M_{rd} are axial and bending capacity of the cross-section, b is the section width, h is the section height, and f_{cd} is the designed compressive strength of FRC. Curves designed using dimensionless parameters from Equations 4 and 5, are called the interaction curves for FRC. They are independent from mechanical characteristics of FRC, dimensions of the cross-section. Interaction curves are derived for different relationships of depth / height of the cross-section (d_1/h), for symmetrically and unsymmetrically reinforced cross-sections, for different reinforcements grades and different fibre properties. In such a manner, FRC interaction curves can be constructed for the cross-sections without conventional reinforcement, by applying the described procedure. Example of such a derived interaction curve is shown in Figure 9.

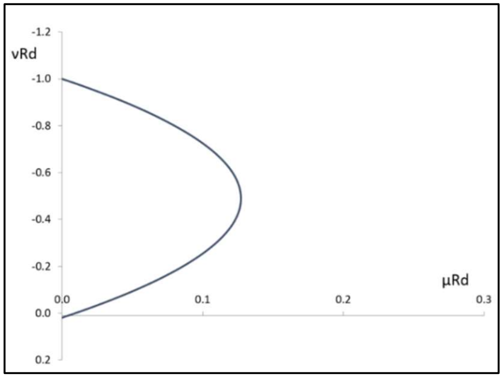


Figure 9. ULS interaction curve for FRC

6.2 Interaction curves for SLS

In the same manner as described in Chapter 5.1, interaction curves for SLS can be derived[11]. Linear stress–strain relationship is assumed, while the compressive stress in FRC for SLS is limited to 60 % of characteristic compressive strength of FRC. Limiting tensile strain in FRC is equal to:

$$\varepsilon_{max} = \frac{w_d}{l_{cs}} \tag{6}$$

Parameter w_d represents maximal crack width, and it depends from the type of FRC, exposure class, construction use etc. Range of the parameter is between 0.15 and 0.3 mm, while full description of the parameter is given in Table 7.6-1 in MC 2010. Hence, in order to successfully design FRC cross-section, pair of the values of axial force and bending moment have to satisfy both ULS and SLS, or, the point defined by those two values have to be inside of the area defined by both interaction curves. Example of SLS interaction curve, for the same parameters used for ULS interaction curve in Figure 9, are shown in Figure 10. By comparing these two Figures, it can be seen that SLS and limitation of the FRC parameters to lower values significantly decrease the area under the interaction curve and hence significantly reduced the amount of possible combinations of axial force and bending moment, which will not lead to the failure of the cross-section.

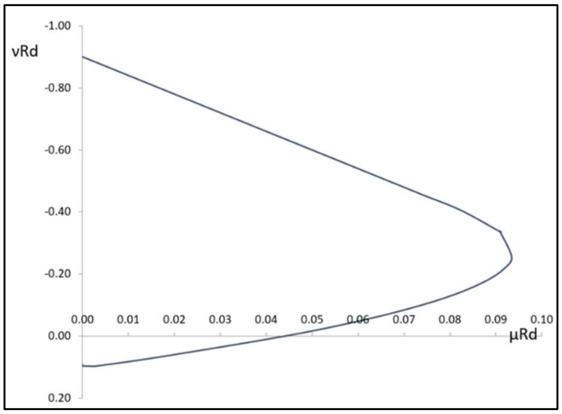


Figure 10. SLS interaction curve for FRC

7. *ANALYSIS*

7.1 Comparison of FRC and RC on an example of reservoir design

Example of the application of the interaction curves for FRC and comparison of FRC and RC is shown on an example of reservoir design. Reservoir is made of the roof slab, supported by the combination of the wall placed at the edges and columns, as shown in Figure 11, where all the necessary parameters are defined.

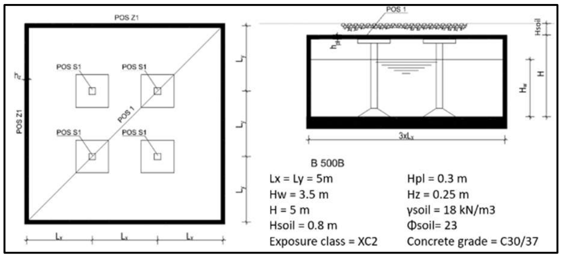
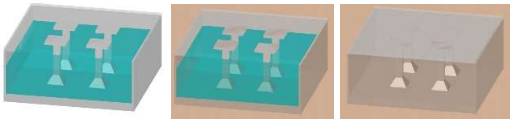


Figure 11. Geometrical and mechanical properties of the designed reservoir

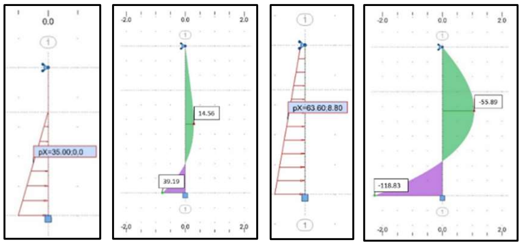
Different loading situations are taken into account: i) Trial charging of the reservoir with water, during which there is no soil pressure (Figure 12a); ii) Phase of the exploitation – Soil pressure and water combination (Figure 12b); iii) Phase of repair, in which there is only soil pressure (Figure 12c). Loading during the phases (i) and (iii) are shown in Figure 13a and Figure 13c, respectively. Values of bending moments are shown in Figure 13b for phase (i) and in Figure 13d for phase (iii).



a) b) c)

Figure 12. a) Phase (i) = Trial charging; b) Phase (ii) = exploitation; c) Phase (iii) = Repair of reservoir

The calculations are made in Autodesk Robot software [7]. For the clamped cross-section at the wall bottom, external side of the value is under tension in phase (i), while internal wall side is under the tension in phase (iii). Axial force reaction in the phase (i) is equal to 15 kN/m while in the phase (iii) it is 43.8 kN/m.



a) b) c) d)

Figure 13. a) Loading during the trial charging; b) Bending moment during the trial charging; c) Loading during the repair; d) Bending moment during the repair

Calculated values of the bending moments and axial force are used to calculate reinforcement according to MC 2010 for RC. In the clamped cross-section, reinforcement near the external edge is A_{s2}=19.59 cm^2/m, and on the internal side of the clamped cross-section A_{s1}=5.57 cm^2/m, acording to ULS analysis. Reinforcement is calculated using Eurocode 2 [12] Equation:

$$A_{s1} = \omega * b * d * \frac{f_{cd}}{f_{yd}} - \frac{Ned}{f_{yd}} \qquad (7)$$

Calcuated crack – width for such adopted cross-section is 0.51 mm which is significnatly higher than allowed 0.15 mm. Hence, additional reinforcement at the inner side of the cross section is adopted A_{s1}=11.05 cm^2/m. For the design of FRC reservoir, FRC 30 3a is assumed. Material parameters of FRC 30 3a are: *fck*=30

MPa, f_{R1}= 3 *MPa*, f_{R3}=0.5*3=1.5 *MPa.* Using previosly derived interaction curves, and assuming the reinforcement calculated for ULS of RC (A_{s2}=19.59 cm^2/m; A_{s1}=5.57 cm^2/m), design of FRC using interaction curve is shown (Figure 14). Value of the bending moment is 1.35 * 118.83 = 160.5 kNm/m ; while axial force is 1.35 * 43.8 = 59.1 kN/m. The coefficient 1.35 is taken from Eurocode 1 [13] for dead load.

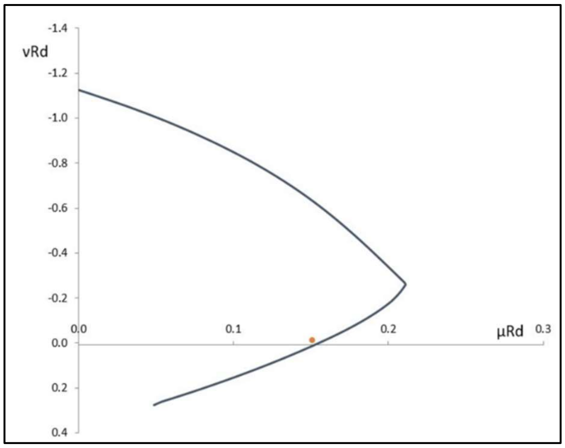


Figure 14. Interaction curve of FRC for ULS with the marked pair of bending moment and axial force in the cross-section

The same procedure is done for the SLS. As previously described, for RC reinforcement had to be increase from 5.57 to 11.05 cm^2/m, due to the limitation of the crack width. As this reinforcement is near the inner edge of the cross-section, and no partial coefficients are used in SLS analysis, value of the bending moment is 39.2 kNm/m, and appropriate axial force is 15 kN/m. By assuming crack width of 0.15 mm (maximal allowed), and using the procedure described in Chapter 5.2, interaction curve is designed for the cross-section regarding SLS and it is shown in Figure 15. The pair of the bending moment and the axial force is marked as an orange dot.

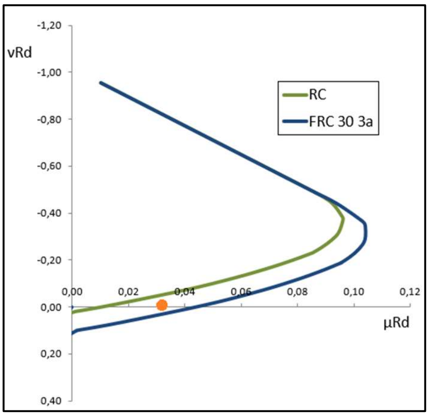


Figure 15. Interaction curve of FRC for SLS with the marked pair of bending moment and axial force in the cross-section

From the Figure 15, it can be seen that FRC capacity satisfies SLS with adopted reinforcement of 5.57 cm^2/m, while in RC it had to be increased to 11.05 cm^2/m. Iteratively, using the interaction curve and successively increasing the bending moment, it is derived that maximal bending moment that FRC cross-section with adopted reinforcement can withstand is 56.5 kNm/m, while RC cross-section can withstand the bending moment of 57 kNm/m, once the increase reinforcement is used. Therefore, it can be concluded that by using FRC, same capacity of the cross-section is reached with 5.57 cm^2/m of reinforcement, as with 11.05 cm^2/m of reinforcement for RC.

8. CONCLUSION

Improvement of concrete behaviour under tensile stresses led to the development of FRC. FRC is a composite material made of concrete matrix with the addition of fibre. It is characterized by the residual tensile strength, which prevents brittle failure of concrete and improves it`s tensile behaviour. Few existing code provisions analyzed the behaviour of FRC and proposed appropriate stress–strain relationships. Yet, lack of the interaction curves for combined bending moment and axial forces is noticed. Due to that, in the scope of the paper, interaction curves are derived based on fib MC 2010 code provision. Interaction curves are derived for ULS, but also for SLS, which represents significant novelty when comparing to RC, for which no such interaction curves are proposed. Therefore, by using derived interaction curves for ULS and SLS for the design of FRC cross–section, calculation time is significantly reduced. Moreover, different values, such as amount of reinforcement or fibre properties can be varied, in order to find optimized solution for the adopted cross–section. Such designed interaction curves are used in order to compare a design of concrete reservoir made of RC and FRC. This example shows the benefit of both, FRC and interaction curves. Using interaction curves, fast design of FRC cross-section is done. Furthermore, bending moment capacity of cross-section according to SLS is derived, for the adopted parameters. When comparing RC and FRC, it is noticed that twice higher amount of the reinforcement at the inner side of the clamped cross section is necessary for RC in comparison to FRC in order to satisfy SLS. Moreover, twice lower amount of reinforcement in FRC provided the same bending moment capacity as RC. Therefore, it can be concluded that FRC is appropriate to use instead of RC, once SLS represent the decisive parameter for the design of the element, especially when limiting value of crack width is hard to satisfy. In such a case, FRC should be used, while use of derived interaction curves for FRC in case of SLS can represent significant benefit in order to optimize the process.

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