Contact Problems: Angioplasty and Stent Deployment Modeling

Isailović, Velibor; Kojić, Miloš; and Filipović, Nenad

Abstract: ***Contact problem between two or more solid bodies is still a significant topic in computational mechanics. The contact problem implies deformation of solid bodies that touch each other. Besides deformation of bodies, friction between bodies can also be taken into account. There are many situations in engineering where the contact problem exists. For example, crash tests in automotive industry, coupled engineering parts with clearance, transition or interference fit, mining process machinery, tire rolling, metal forming processes, etc. Many different approaches can be used for solving contact problems (Wriggers 2008). In this paper, we present a simple approach based on modified one-dimensional elements for solving contact problems. That modified one-dimensional elements, so-called elastic supports, we introduce in nodes where the contact appears. Also, when nodes get out of the contact, those elements are deleted from the model. The basic idea is to find positions of boundary nodes which are in contact with another body's boundary nodes (or faces) and to add additional stiffness in equations that correspond to degrees of freedom of that node. This is a general method for solving contact problems and it can be applied to modeling of angioplasty endovascular procedure or modeling of medical stent deployment problem. The results section shows initial results for the angioplasty procedure modeling, obtained by the described methodology.***

Index Terms: ***Contact mechanics, finite element method, angioplasty***

1. *Introduction*

Cardiovascular diseases are the most common cause of death. There are many different diseases in this group. One of them is atherosclerosis or arteriosclerotic vascular disease. During the progression of this disease the plaque builds up inside arteries. The structure of plaque contains fat, cholesterol, calcium and other substances from the blood, smooth muscle cells from intima, etc. Patients with this disease have to be treated by proper medical therapy. But, if a plaque is very large, i.e. the narrowing caused by plaque slows down or stops blood flow, patients have to be subjected to medical interventions like angioplasty or stent implantation [2], [7].

Angioplasty is a clinical method which implies insertion of balloon-tipped catheter inside the blood vessel with narrowing caused by atherosclerotic plaque. With that balloon, cardiologists try to expand the narrowed blood vessel. In case that is not enough, the stent implantation is a better solution. Stents are metal or plastic wired tubes that can be inserted into narrowed arteries to hold them open. Those procedures are similar in terms of mechanics, and it could be very useful to have mechanical model to investigate these processes "in silico". That was the reason why we decided to develop such model.

This paper contains the following sections: Methods, Results and Conclusion. In the Methods section, numerical methodology applied to solving contact problem using elastic supports is described. The Results section shows initial results obtained on the basis of the developed numerical model. The algorithm is applied to solving parametric finite element model of blood vessel with stenosis and elastic balloon which should open stenosis. The Conclusion section gives summary of the developed numerical software and parametric finite element model with reference to the future work.

2. *Methods*

The most commonly used method for solving continuum mechanics' problems is the finite element method. Starting from equilibrium equations [1], [6], [9]:

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V. Isailovic and N. Filipovic are with the Faculty of Engineering, University of Kragujevac, Serbia. M. Kojic is with The Methodist Hospital Research Institute, The Department of Nanomedicine, 6670 Bertner Ave., Houston, TX 77030 and Serbian Academy of Sciences and Arts, 35 Knez Mihailova Street, 11000 Belgrade, Serbia. All authors also have affiliation with Bioengineering Research and Development Center – BioIRC, Kragujevac, Serbia.

$$\frac{\partial \sigma_{ik}}{\partial x_k} + f_i^V = 0, i,j = 1,2,3 \qquad (1)$$

including inertial and dissipative forces and satisfying loading and displacement boundary conditions [6], we can derive the basic equations for solid dynamics in the discrete form:

$$\boldsymbol{M}\ddot{u} + \boldsymbol{B}\dot{u} + \boldsymbol{K}u = \boldsymbol{F} \qquad (2)$$

Where is:

$\boldsymbol{M} = \int_V \rho N^T N dV$ – mass matrix,

$\boldsymbol{B} = \int_V bN^T N dV$ – damping matrix,

$\boldsymbol{K} = \int_V B^T C B dV$ – stiffness matrix,

$\boldsymbol{F}$ – vector of external forces,

$\ddot{u}$ – vector of accelerations,

$\dot{u}$ – vector of velocities,

u – vector of displacements.

In order to solve this equation, the Newmark's procedure can be used. The basic approximation used in the Newmark method is that the acceleration within the time step is considered constant, and is given as:

$$\ddot{u}(\tau) = (1-\delta)\, {}^{n}\ddot{u} + \delta^{n+1}\ddot{u}, 0 \leq \tau \leq \Delta t \qquad (3)$$

Where $0 \leq \delta \leq \Delta t$ is a parameter; $\delta = 0$, $\delta = 1$, and $\delta = 0.5$ correspond, respectively, to the Euler forward, Euler backward, and trapezoidal integration scheme (Fig. 1).

Integrating this equation with respect to time τ, the velocity ${}^{n+1}\dot{u}$ and displacement ${}^{n+1}u$ at end of time step are obtained as:

$${}^{n+1}\dot{u} = {}^{n}\dot{u} + \left[(1-\delta)\, {}^{n}\ddot{u} + \delta^{n+1}\ddot{u}\right]\Delta t \qquad (4)$$

and:

$${}^{n+1}u = {}^{n}u + \frac{1}{2}\left[(1-\delta)\, {}^{n}\ddot{u} + \delta^{n+1}\ddot{u}\right](\Delta t)^2 \qquad (5)$$

In order to improve the solution accuracy and stability [1], instead of previous equation the following expression for the displacement is used:

$${}^{n+1}u = {}^{n}u + {}^{n}\dot{u}\Delta t +$$

$$\frac{1}{2}\left[(1-\delta)\, {}^{n}\ddot{u} + \delta^{n+1}\ddot{u}\right](\Delta t)^2 \qquad (6)$$

where is α another integration parameter. It can be shown that the best solution accuracy is obtained for $\delta = 0.5$ and $\alpha = 0.5$. Now we substitute ${}^{n+1}\ddot{u}$ from (6) to (5) and express ${}^{n+1}\ddot{u}$ in terms of the displacement ${}^{n+1}u$ as:

$${}^{n+1}\ddot{u} = \frac{1}{\alpha(\Delta t)^2}\left[{}^{n+1}u - {}^{n}u - {}^{n}\dot{u}\Delta t - \left(\frac{1}{2} - \alpha\right)(\Delta t)^2\, {}^{n}\ddot{u}\right]$$

Then, by substituting this expression for ${}^{n+1}\ddot{u}$ into (5) it follows that:

$${}^{n+1}\dot{u} = \frac{\delta}{\alpha\Delta t}\left({}^{n+1}u - {}^{n}u\right) - \left(\frac{\delta}{\alpha} - 1\right){}^{n}\dot{u} - \left(\frac{\delta}{2\alpha} - 1\right)\Delta t\, {}^{n}\ddot{u}$$

Then we substitute the expressions for ${}^{n+1}\ddot{u}$ and ${}^{n+1}\dot{u}$ in the differential equation of motion of the finite element:

$$\boldsymbol{M}^{n+1}\ddot{u} + \boldsymbol{B}^{n+1}\dot{u} + \boldsymbol{K}^{n+1}u = {}^{n+1}\boldsymbol{F}^{ext} \qquad (7)$$

and obtain the system of algebraic equations:

$$\widehat{\boldsymbol{K}}^{n+1}u = {}^{n+1}\widehat{\boldsymbol{F}}^{ext}$$

where are:

$$\widehat{\boldsymbol{K}} = \boldsymbol{K} + a_0\boldsymbol{M} + a_1\boldsymbol{B} \qquad (8)$$

$${}^{n+1}\widehat{\boldsymbol{F}}^{ext} = {}^{n+1}\boldsymbol{F} + M\left(a_0\, {}^{n}u + a_2\, {}^{n}\dot{u} + a_3\, {}^{n}\ddot{u}\right) + B\left(a_1\, {}^{n}u + a_4\, {}^{n}\dot{u} + a_5\, {}^{n}\ddot{u}\right)$$

The coefficients in previous equation are:

$a_0 = \frac{1}{\alpha\Delta t}$, $a_1 = \frac{\delta}{\alpha(\Delta t)^2}$, $a_2 = \frac{1}{\alpha\Delta t}$

$a_3 = \frac{1}{2\alpha} - 1$, $a_4 = \frac{\delta}{\alpha} - 1$, $a_5 = \left(\frac{\delta}{2\alpha} - 1\right)\Delta t$

Since we imposed the condition of satisfying the differential equations at the end of the time step, the Newmark method is thus implicit.

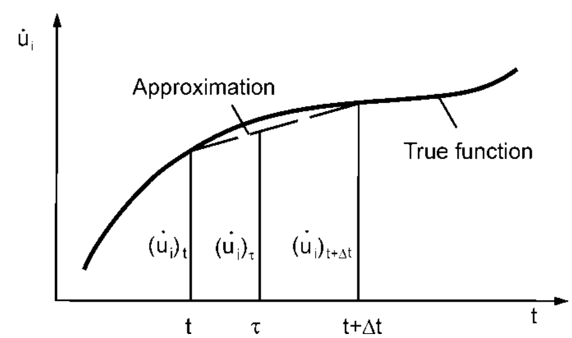

Figure 1: Approximation for velocity of a material point within a time step

The numerical software based on equation (2) can be used to solve a wide range of problems in the field of solid mechanics. For example, this software can be used for modeling deformation of solid structures under prescribed loads or prescribed displacements on some boundary of the model. Moreover, the software can be used for modeling solid body motion (solid dynamics). But, if we want to model interaction of two or more solid bodies, it is necessary to develop an algorithm which could be able to recognize the contact of two or more bodies, and ensure no overlapping between the bodies. There are several methods for solving this kind of problems [8], but we will propose quite simple method based on one-dimensional finite elements.

Contact problem can be solved using elastic supports – finite elements similar to one-dimensional finite elements. One-dimensional finite elements have only two nodes and six degrees of freedom: three translations in both nodes (Fig. 2).

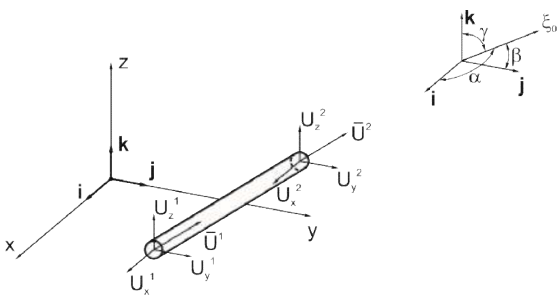

Figure 2: One-dimensional finite element with two nodes and six degrees of freedom

The finite element equation for single one-dimensional element has the following form [6]:

$$\overline{K}^e U^e = F^e$$

where:

$\overline{K}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ - stiffness matrix in local system,

$\overline{U}^e = \begin{Bmatrix} \overline{U}^1 \\ \overline{U}^2 \end{Bmatrix}$ – vector of nodal displacements in local coordinate system,

$\overline{F}^e = \begin{Bmatrix} \overline{F}^1 \\ \overline{F}^2 \end{Bmatrix}$ – vector of nodal forces in local coordinate system.

These three equations can be transformed to the global Cartesian coordinate system using the following transformation matrix:

$$T = \begin{bmatrix} \cos\alpha & \cos\alpha\beta & \cos\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \cos\beta & \cos\gamma \end{bmatrix}$$

In Fig. 3 one-dimensional element and elastic support are shown. They are very similar, but there are several significant differences. The first difference is the direction of the element. In the case of one dimensional elements, if one node is fixed, that element can have free rotation around that node. Hence, the direction of the element is not constant. In the case of elastic supports, the direction of the element does not depend on the motion of the node where the element is placed, it is always constant. The second difference is the ability to support the node the element is connected to, only in the case when the element is subjected to the pressing load but not to the tensile load.

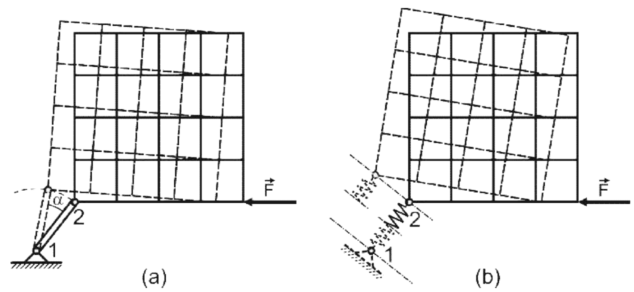

Figure 3: One-dimensional finite element (a) and elastic support (b)

Fig. 4 shows mechanism of interactions between two bodies [3], [4]. We observe nodes from one solid body and outer faces of another body. When a node from the boundary of the first body comes into contact with the outer surface of the second body, new elements appear in the finite element model – elastic supports. The direction of elements is normal to the body surface in contact point. The stiffness of elastic support should be greater than stiffness of solid bodies. In general, it should be significantly higher, because it can cause some numerical problems and instability. When the contact between bodies disappears, elastic supports also disappear. Hence, for each time step we search for contact nodes, if they exist we add elastic supports in the system of equations and remove them at the end of the current time step.

This procedure applies to both bodies, because outer nodes of the first body can come in contact with faces of the second body and vice versa, the nodes of the second body can come in contact with outer faces of the first body.

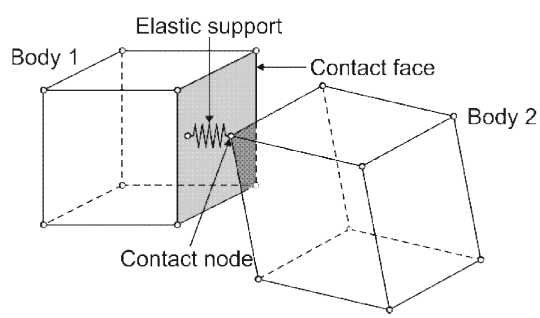

Figure 4: Interaction between solid bodies: two bodies in contact and elastic support placed in contact point

The proposed algorithm is general and can be applied to any type of contact problems. Also, nonlinearities like nonlinear material models or geometrically nonlinear behavior of model can be simulated along with contact problems.

3. Results

In this section, we will present initial results for contact problem obtained with solver developed under PAK software package [5].

The initial model is parametric model of blood vessel with stenosis and balloon which should open stenosis. This model contains 4182 nodes and 2560 three-dimensional 8-node linear finite elements. Materials of both structures are linearly elastic, but with large deformations. Boundary conditions applied in this model are fixed nodes at the beginning and at the end of the balloon and blood vessel with stenosis, and prescribed pressure in the balloon. Value of the pressure is not significant in this case, because it is fitted only to open stenosis. The pressure increases during time linearly. Simulation has 200 time steps of 5 milliseconds.

Besides the mentioned number of 3D finite elements, when the balloon and stenosis are in contact, there are a variable number of elastic support elements, which depends on the contact area size. In the beginning of the contact, there are only few elements in the region around stenosis, but with time, the pressure in the balloon increases, the area of contact also increases and, consequently, the number of elastic supports. After some time, the contact also appears on the side opposite to the stenosis and, together with that, new elastic supports also appear.

In Fig.5 results from some time steps during simulation are shown. In Fig.5A displacement field in step 50 is shown. That step is the first step when the contact between the balloon and stenosis occurs. In Fig.5B results from time step 100 are shown. In this step, displacements of wall around stenosis are visible. Fig.5C shows displacement field in the model in time step 150 when most parts of the outer balloon surface and inner blood vessel surface are in contact. In Fig.5D, the last step of simulation is presented. Here, the blood vessel with stenosis is completely open.

4. Conclusion

In this paper, we proposed a simple way to develop contact algorithm and apply it to modeling angioplasty and stent deployment processes. Modeling of these processes has a significant role in investigations and development of stents and equipment for angioplasty and stent deployment.

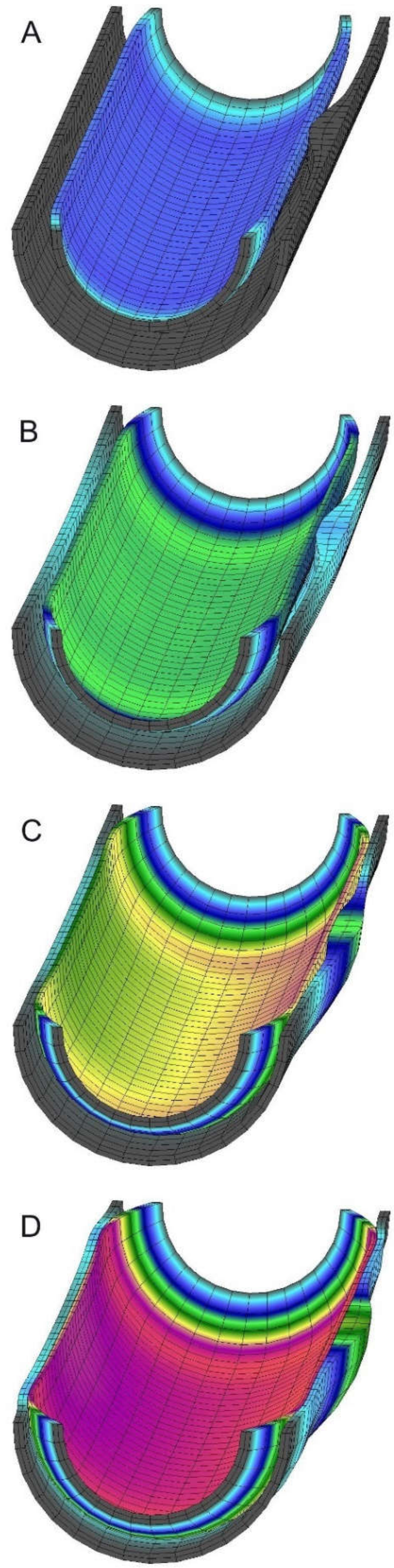

Figure 5: Parametric model of inflating balloon inside blood vessel with stenosis

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