Potential field-based approach for obstacle avoidance trajectories

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Abstract: This paper proposes a new mathematical description of the potential field used in obstacle avoidance trajectory design. The main benefits of this description are the quickness of minimum computation and the compensation for the main drawbacks of potential field method. After the presentation of the potential field definition and its minimum computation this concept is included into an obstacle avoidance trajectory design method expressed under the form of an obstacle avoidance trajectory algorithm. A state-space controller is designed in order to control the car on the obstacle avoidance trajectory. Digital simulations performed for a complete dynamic model of a car validate the method.

Index: Obstacle avoidance, potential field, trajectory design, control, simulation.

1. INTRODUCTION

In the field of obstacle avoidance trajectory, several methods are known. These methods can be classified in the following ways. From the avoidance manoeuvre concept point of view the classification includes the potential field method, the vector histogram method, the curvature-velocity method and also methods based on artificial intelligence (AI) tools including the fuzzy logic approach. From the point of view of the road map decomposition these methods can be divided into global and local ones.

The potential field method is based on the artificial elastic mesh construction according to the results presented in [3,4,5,13,14,15]. This mesh incorporates obstacles and car trajectory. More precisely, the obstacles generate repulsive forces on the car and the trajectory generates attractive forces. The equilibrium between reactive and attractive forces deforms the desired trajectory and transforms it into an obstacles avoidance trajectory. The method was developed for static or even mobile obstacles. From this concept the elastic band concept is derived [11] which considers that the avoidance trajectory is the deformed shape of an elastic band (shape deformed by the obstacles). Another use of potential field is the velocity obstacle approach, where the potential collision is computed employing an obstacle motion prediction [8]. The histogram approach suggested in [2] computes both the motion direction and the velocity from the transformation of the occupancy into a histogram description.

The curvature-velocity method analyzed in [7, 9] is based on the assumption that the vehicle moves in circular or linear paths. The motion commands are searched directly in the space of possible linear and circular velocity. The concept of possible velocities is laid with the concept of dynamic window. In the end, to find the appropriate combination of linear and rotational velocities (included in the dynamic window) a minimization problem is solved. The method is developed by wave propagation techniques in the dynamic window [16]. Fuzzy logic methods have been developed and discussed in [6, 19] because the obstacle avoidance means navigation in uncertain environments, and these methods can deal with vague, imprecise and uncertain information. The well-known robustness of these methods is another reason that motivates these trails. If the road map decomposition point of view is accepted, the global methods assume that a complete model of the environment is available; this means that a complete trajectory from the starting point to the target can be either computed online or stored in memory. Local approaches use a small fraction of the world model in order to generate the control signals.

Because the method to be proposed here belongs to the category of potential field methods, it possesses the well-acknowledged drawbacks of these methods [12, 28]: trap situations due to local minima; no passage between nearness obstacles; oscillations in the presence of obstacles; oscillations in narrow passages. A different variant of these methods is to divide the avoidance manoeuvre in two steps, planning the avoidance trajectories and next control the vehicle on these trajectories. Such an algorithm is proposed in [3], and it is based on the computation of the trajectories from the sum of repulsive and attractive force; a PID controller is
tuned such that to control the vehicle on the avoidance trajectory. From the authors’ point of view this method has the following shortcomings: the equilibrium position computation is a time consuming process; in real-world applications the shape and the dimensions of the obstacles are discovered during the avoidance manoeuvre (so it is impossible to offer a global solution), and just a PID controller does not seem to be enough robust to cope with the dynamic model of the car.

This paper extends the previous theoretical results reported in [28]. Another application is included and described shortly.

The paper is organized as follows. The next section starts with presenting the main concepts in obstacle avoidance. Then, the minimum potential field concept is profoundly studied from the problem definition up to the mathematical model involved. Section 3 is focused on an original obstacle avoidance trajectory algorithm based on the previous potential field definition. Simulations for a car example are presented in Section 4, and Sections 5 ends the paper with conclusions.

2. THE MINIMUM POTENTIAL FIELD

The definition of the task to be accomplished should be presented firstly. The robot is following a trajectory, which is a priori defined in the map and in a certain moment, discovers an obstacle that makes the initial trajectory impossible to pursuit. The car control system must react and fulfill the following online tasks: discover the shape and position of the obstacle; compute the avoidance trajectory; control the car on the trajectory; return to the initial trajectory.

Since the obstacle is discovered online, during the avoidance manoeuvre, the avoidance trajectory will be composed by several parts; this means that, online, the control system design must make use of a local approach as a loop composed by: discovering the obstacle and the road (the universe), designing a part of the avoidance trajectory and controlling the car on this trajectory part. Because this is an online process, any time consuming computations must be avoided. For this reason, it is needed to link together from a mathematical point of view the knowledge about the universe with the creation of the trajectory. This means that it is important to consider the sensor behaviours in the design of the obstacle avoidance trajectory algorithm final stage.

The analysis of some actual approaches concerning the potential field method [3,4,5,13,14,15] leads to the conclusion that the avoidance trajectory represents the static deformation of a hypothetical elastic network associated with the robot, road and obstacle. It is also known that the equilibrium of these kinds of structures can be obtained in terms of imposing a minimum potential energy condition.

The new approach starts with the idea stating that it is not necessary to construct such a complicated potential field and compute the numerical solution of equilibrium. In fact, it is more suitable to construct a simple potential field, which can be used (minimized) relatively quickly. The construction of the potential field is based on finding a mathematical function having the following properties:

- in the absence of the obstacle the minimum of this function is the initial trajectory, the presence of the obstacle will generate a new minimum to avoid the obstacle,
- after the obstacle avoidance the trajectory must converge to the a priori one,
- the function definition can be connected to the relative speed between the obstacle and the car,
- the function can be constructed easily employing information from sensors data and initial trajectory data,
- the computation of the minimum must be less time-consuming process.

In order to present the mathematical function of the potential field, the potential of the road in absence of obstacles is presented. First a mesh is defined on the road (in the Oxy plane) and for each grid a potential $P_R$ is defined according to the mathematical function (1):

$$P_R = z(x, y), \ (x, y) \in (x_{1..n}, y_{1..m}).$$ (1)
The graphical representation of this function is illustrated in Figure 1a. The function has two maximums linked to the road margins and a minimum to the desired trajectory of the car. Because the potential function is discrete, several slices \( P_k = z(x_i, y_i), \) \( i = \overline{1, m} \) can be considered. The potential minimum of the slice is a point on the desired a priori trajectory.

In the end, the potential function minimum is a collection of \( m \) points. Because the obstacles are discovered in the car referential system, the detailed expression of the function (1) must be defined here and it must preserve the a priori trajectory. Figure 1b illustrates the link between the road and the car referential systems.

The mathematical expression of the function in (1) expressed in the car referential system is (2):

\[
z(x, y) = \frac{|y - a(x, y)|}{2} m_2 (\text{sgn}(y - a(x, y)) + 1) - m_1 (\text{sgn}(y - a(x, y)) - 1)
\]

where:

\[
a(x, y) = \frac{X_o - A(y) - x \sin \psi}{\cos \psi},
\]

\( A(y) \) is the desired position of the car in the road referential frame, \((X_0, Y_0)\) is the car position on the road referential frame, \( \psi \) is the car orientation, \( L \) is the width of the road.

Since the use of a laser scanner is intended, the obstacles are recognized like collections of points in the Oxy plane. More precisely, such a point is defined by the distance and angle between the car (scanner) referential and the scanned point belonging to the object. The obstacles potential is constructed for the same mesh grid \((x_{i,n}, y_{1,m})\) already defined for the road potential function. The idea is to generate a potential function for each scanned point, so in the end the obstacle potential function will be a sum of potential functions. The mathematical model of the obstacle potential function \( P_O \) is:

\[
P_O = \sum_{i=1}^{k} e^{-\frac{(x - y)^2 + (x - x)^2}{2\sigma}}
\]

where: \( c_1, c_2 \) and \( \sigma \) are the parameters linked to the steering motor performance and the relative speed between the car and the obstacle, and \((x_i, y_i)\) is the position of the \( i \)-th scanned point with index \( k_i \), \((x_i, y_i) \in (x_{i,n}, y_{1,m})\). The object identification is not generated and the scanned points are used directly. Figure 2a illustrates such a potential function for three scanned points \((k = 3)\).

If the two potential functions defined here will be added, the universe (road and obstacles) potential function \( P_U \) will be obtained according to (5):

\[
P_U = P_R + P_O.
\]

The avoidance trajectory at the first step will be defined like a collection of points (6):

\[
\Gamma = \{(x_i, y_i)_{\min} \mid i = \overline{1, n}\}.
\]

This collection of points is computed with the minimum of the universe potential function. In order to obtain this minimum, the optimization problem (7) must be solved:

\[
y_i_{\min} = \arg \min_y \ P_U(x, y)
\]

subject to \( y \in \{y_{1,m} \mid y_{i-1} - I < y_{i-1} < y_{i-1} + I\} \)

\[
\text{subject to } y \in \{y_{1,m} \mid y_{i-1} - I < y_{i-1} < y_{i-1} + I\} \interrightarrow y_{i-1}^\text{min} + I
\]

where \( I = \text{const} \) represents the vicinity radius. Several numerical methods can be employed to solve this problem [22, 27]. The computational aspects related to (7) are not time-consuming because the potential field is divided into slices and for each slice the computation of the minimum consists in finding the smallest element of a vector. The search area is constrained to \( y_i \in (y_{i-1}^\text{min} - I, y_{i-1}^\text{min} + I) \) in order to avoid the jump of the minimum point from a local minimum, which is in the vicinity of the previous minimum, to a global minimum. The reasons of this constrained are:
• there are cases when the global minimum is a shaded point (in the obstacle back), so choosing this point means passing through the obstacle (see Figure 2a),
• even if the global minimum point is not a shaded point, these jumps of minimum point outside the vicinity \( y_{j} \in (y_{i-1}^{\text{min}} - I, y_{i-1}^{\text{min}} + I) \) will generate a rugged trajectory.

Two strategies can be imagined characterized by either using (directly) the minimum point in the command decision or split the problem in two steps, first designing the avoidance trajectory and second, controlling the car on this trajectory. The second version is chosen here but the first one is also very attractive to be developed in a future work. With (7) a collection of minimum potential points is obtained. Based on this result a smooth curve is designed, and this curve will be the avoidance trajectory. Some comments with this regard are:
• a \( C_2 \) class curve must be designed because the car can follow (without slippage) linear, circular or clotoidal trajectories,
• because the obstacle is discovered during the avoidance manoeuvre, only a part of these points and will be used to define a part of the avoidance trajectory,
• the avoidance trajectory will be composed by several parts.

This idea is illustrated in Figure 2b.

In order to define the mentioned part of the trajectory the following steps are necessary:
• from the minimum points, select the first \( k \) and compute the middle point \((x_{\text{mid}}, y_{\text{mid}})\),
• define the direction \( t \) using the middle point and point \( k \),
• corroborate the initial position, direction (tangent to Ox) and curvature (0) with the final position \((x_{\text{mid}}, y_{\text{mid}})\), direction \( t \), curvature (0) and define a spline trajectory.

For each trajectory part a polynomial function will be obtained:

\[
y(x) = a_5x^5 + a_4x^4 + ... + a_0 , \quad (8)
\]

where \( a_{0,5} \) are found by imposing the boundary conditions (9):

\[
y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0,
\]
\[
y(x_{\text{mid}}) = y_{\text{mid}}, \quad y'(x_{\text{mid}}) = t, \quad y''(x_{\text{mid}}) = 0 . \quad (9)
\]

The following comments are emphasized in this context:
• the number of points \( k \) is related to a certain length \( d \) (Figure 2b), choosing the first \( k \) points means this prediction is trusted enough, so that the trajectory will be followed upon the distance \( d = x_{\text{mid}} \),
• the final trajectory is composed by several parts,
• since each part is a \( C_2 \) curve and the boundary conditions are considered, the final trajectory will be also a \( C_2 \) curve.

This algorithm includes the avoidance trajectory definition and also decisions that must be taken by the control system during this maneuver. The algorithm consists of the following steps:
1. Set the desired trajectory in accordance with the global map of the locomotion. This trajectory is not related to the unknown obstacle.
2. Scan the road permanently and, if no obstacles are found, control the car on the desired trajectory.
3. If obstacles are scanned, compute the avoidance trajectory as follows:
   1. Adjust the coefficients \( c_{1,2} \) in accordance with the relative speed between the car and the scanned points. Compute the potential function in (5).
   2. Compute the minimum of the potential function using (6) and (7).
   3. Choose the first \( k \) points for a predicted trust distance \( d \) and compute the part of the avoidance trajectory using (8).
   4. Control the car on the avoidance trajectory.
   5. If during the locomotion the sensors accept this trajectory from the beginning to the end, go to step 2. Else, continue with 3.6.
   6. If sensors will denial the trajectory, adjust the predicted trust distance \( d \) and go to 3.1. Else, stop.

This algorithm needs sensor data fusion. With this sensor system several variables must be computed including the position of the scanned point and the relative velocity between the robot and scanned point. Moreover, this system must enable the decision making on either accepting or rejecting of the current trajectory part.

In order to simulate the avoidance manoeuvre, a set of Matlab [20] programs has been developed. The scanner behaviour, which discovers this universe, can be simulated and the potential function and its minimum can be computed. All these actions are made for each part of the avoidance trajectory. In the end, the entire image of universe and avoidance trajectory is presented. We note that running these programs, the robot discovers the original unknown universe step by step, and takes decision using local knowledge.

![The scanned points to the car reference frame](image)

A sample of the simulation results is presented in Figure 3 and Figure 4. Since the avoidance trajectory (in this case) is composed by several parts (here 20) only the results corresponding to
step 10 are presented here, and the final result has been inserted. The entire avoidance trajectory is composed by all 20 parts in this case. The plots presented in Figure 3 show the scanned points in the car referential frame and Figure 4 show the potential of road and obstacle.

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3. Conclusion

The proposed obstacles avoidance trajectory methods belong to the potential field methods. The idea was to replace the nonlinear potential functions, which require numerical methods in order to compute its minimum with discrete potential functions subject to very quickly minimization. We consider that the present work improves the potential field methods due to its mentioned mathematical construction and because it integrates directly the sensor data in the mathematical model and it avoids the mentioned oscillations narrow obstacles and passages.

The suggested method has been integrated to an obstacle avoidance algorithm. According to this algorithm, the vehicle discovers on line the universe and takes decision in order to reach its goal. The goal is an a priori trajectory that must be transformed because of the (initial unknown) obstacles. These transformations are done in two steps, first the obstacle avoidance trajectory is designed next the control design on this trajectory is performed. The controller design is based on a state-space method.

Several parts compose the obstacle avoidance trajectory. Each part is related to the knowledge about the local regions. Locomotion on such trajectory part involves a certain trust distance, which is subject to online adjustments.

The algorithm was validated by digital simulation of both the algorithm and the control system behaviour. Future research will be focused on:

- experiments,
- the derivation of a mathematical model based on the car and steering system dynamic in order to compute the coefficients $c_{1,2, \sigma}$,
- the derivation of a mathematical model which will predict the part $n$ of the avoidance trajectory before the car finishes the part $n-1$ of the avoidance trajectory,
- the incorporation of advanced control strategies to improve the control system performance indices, accompanied by several analyses [1, 24, 25, 26].

Acknowledgment

The paper is supported by the Romanian Ministry of Education, Research and Innovation through the PNII Idei project 842/2008.

The support of HUNOROB project (HU0045, 0045/NA/2006-2/OP-9), a grant from Iceland, Liechtenstein and Norway through the EEA Financial Mechanism and the Hungarian National Development Agency, a Széchenyi István University Main Research Direction Grant, and National Scientific Research Fund Grant OTKA K75711 is also acknowledged.

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